KNT/KW/16/5041

Bachelor of Science (B.Sc.) Semester—I (C.B.S.) Examination

MATHEMATICS

(M_1 : Algebra and Trigonometry)

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Find the rank of the matrix :

$$A = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 1 & 2 & 0 & -1 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

by reducing it to the normal form. 6

(B) Show that the equations :

$x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y - z = -1$ are consistent and hence solve them. 6

OR

(C) Find the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 6

(D) Verify that the matrix :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 3 & -1 & 1 \end{bmatrix}$$

satisfies Cayley-Hamilton theorem. 6

UNIT–II

2. (A) Solve the equation $x^3 - 3x^2 + 4 = 0$, given that two of its roots are equal. 6

(B) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in the geometrical progression. Hence or otherwise solve the equation $x^3 - 7x^2 + 14x - 8 = 0$. 6

OR

(C) Solve the reciprocal equation :

$$x^5 - 5x^4 + 6x^3 - 6x^2 + 5x - 1 = 0$$

by reducing it to the standard form. 6

(D) Solve the biquadratic equation :

$$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$$

by Ferrari's method. 6

UNIT–III

3. (A) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then prove that :

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma) \text{ and}$$

$$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma).$$ 6

(B) Prove that n, n^{th} roots of unity forms a series in geometric progression. 6

OR

(C) If $\sin(A + iB) = x + iy$, then prove that :

$$\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1 \text{ and}$$

$$\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1.$$ 6

(D) Prove that :

(i) If $\cosh y = x$, then

$y = \cosh^{-1} x = \log [\, x + \sqrt{x^2 - 1}\,]$ and

(ii) If $\sinh y = x$, then

$y = \sinh^{-1} x = \log [\, x + \sqrt{x^2 + 1}\,]$. 6

UNIT-IV

4. (A) Let G = (G, 0) be a group. Then prove that :

(i) $(a^{-1})^{-1} = a, \ \forall \ a \in G$

(ii) $(a \circ b)^{-1} = b^{-1} \circ a^{-1}, \ \forall \ a, b \in G$. 6

(B) Show that the set G = {1, 2, 3, 4} is an abelian group of order 4 with respect to multiplication modulo 5. 6

OR

(C) Prove that the intersection of two subgroups of a group G is a subgroup of G. Give an example to show that union of two subgroups of a group G is not necessarily a subgroup of G. 6

(D) Prove that every permutation can be expressed as a product of transpositions. Write the permutation :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$$

as a product of transpositions. 6

QUESTION–V

5. (A) Write the augmented matrix for the system of equations :

$y + 2z = a, \ x + 2y + 3z = b, \ y + z + 3x = c$. 1½

(B) Write the characteristic equation of the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$. 1½

(C) Form the equation whose roots are 3, –1, 5. 1½

(D) Show that the equation $x^4 - 2x^3 - 1 = 0$ has at least two complex roots by using Descarte's rule of sign. 1½

(E) Show that sin (iz) = i sinh z. 1½

(F) Show that Log (xi) = logx + i (2n + ½)π. 1½

(G) Find the cycles and orbits of the permutation :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 2 & 1 & 6 & 5 & 7 \end{pmatrix}$$ 1½

(H) H = {1, –1} is a subgroup of the multiplicative group G = {1, –1, i, –i}. Show that group G is the union of disjoint right cosets of H in G. 1½